

K34: Climate Modelling

Designing and running computer models of the physics of a climate system to predict (or post-dict) how climate will change under differing assumptions

A Simple Zero-Dimensional Model

$$(1 - a)S\pi r^2 = 4\pi r^2 \epsilon \sigma T^4$$

- **Heat in = Heat out, in equilibrium.** We get heat from the sun, and we radiate heat back out into space. If the incoming solar heating is unchanged, the effective equilibrium temperature of the Earth must be unchanged too.
- This simple model assumes the Earth is perfectly uniform across its entire surface, receives the same heating per square foot from the sun at all locations and so there is no heat flow within the Earth system, neither across the surface, nor vertically in the ocean and atmosphere
- This model is too oversimplified to account for climate and temperature change, for several reasons we'll examine. First note...
- Averaged over the day and night side, including albedo, clouds... etc., the Earth receives 240 W/m² from the sun. For Climate to be stable, the Earth needs to radiate 240 W/m² back out to space

But the Earth is non-uniform.

Oceans, Deserts, Cities...

- The Earth radiates not at a fixed place (the surface, for example), it radiates some energy from the surface and some at a wide range of altitudes in the atmosphere – so you can have more energy coming from some layers and less from others and thereby see both rising temperatures and also falling temperatures, at different altitudes
- Also, the simple model applies only averaged over time. The sun does change its luminosity slightly with the solar cycle, and a given place on earth goes through a daily and seasonal heating cycle as well.

And, The Earth is NOT in equilibrium with the current greenhouse gas content of our atmosphere.

- We radiate 0.58 watts/m² LESS than we receive from the sun, (more recently data says 0.83 watts/m²)
- This is because we continue to add CO₂ and other GHG's to the atmosphere, and the Earth can't "catch up" fast enough to keep up with our new CO₂ levels.
- Therefore, we will continue to heat up until we are indeed in equilibrium.

But Still - the Zero-Dimensional Model is Educational...

- Solving for the Temperature gives...

$$T = \sqrt[4]{\frac{(1 - a)S}{4\epsilon\sigma}}$$

- For measured albedo $a=0.3$ (i.e. Earth reflects 30% of incoming solar energy), and idealized emissivity of 0.612, and incoming solar constant of 1367 W/m^2 we get then...
- ...An effective radiation temperature of the Earth = **288K = 59F**

A more careful calculation...

- ...raises the emissivity of the Earth, after realizing that clouds cover much of the planet and have an average emissivity of 0.5, while land has emissivity of 0.7-0.99
- Higher emissivity means a cooler Earth; resulting equilibrium temperature then is $285\text{K} = 53\text{F}$. **Not a bad estimate, for such a simple model!**
- But, a zero dimensional model doesn't account for movement of heat around the Earth and won't tell us about current global warming

The Basic Equations of Climate

- The starting **basic equations** are a set of nonlinear differential equations that are used to approximate global atmospheric flow. They consist of three main sets of equations describing physical law:
- ***Fluid motion under Forces, with Conservation of Momentum***: Consisting of a form of the Navier-Stokes equations, specialized to describe fluid flow on the surface of a sphere under the assumption that vertical motion is much smaller than horizontal motion (hydrostasis) and that the fluid layer depth is small compared to the radius of the sphere
- ***A Thermal Energy equation*** relating the temperatures of the system to heat sources and heat sinks, including the heat transfer modes we talked about
- ***A continuity equation***: Representing the conservation of mass.

Grit Your Teeth, and Don't Avert Your Eyes.... And Just Endure the Next Few Slides.

- I know these mathematics are beyond any required pre-req's for this course,
- Jam a stick between your teeth and just look anyway – for the good of your soul!
- I'll try to go quickly

Pressure and Force

- A Pressure gradient dp/dx across a fluid element results in a net force f on each fluid element of mass m and density ρ

$$\frac{f}{m} = \frac{1}{\rho} \frac{dp}{dx}$$

- The force f due to viscous friction within the fluid

$$f_r = \frac{f}{a} \frac{1}{\rho} \mu (\nabla \cdot (\mu \nabla v) + \nabla (\lambda \nabla \cdot v)).$$

East-West Winds, North-South Winds, Thermodynamics Equation

$$\frac{\partial u}{\partial t} = \eta v - \frac{\partial \Phi}{\partial x} - c_p \theta \frac{\partial \pi}{\partial x} - z \frac{\partial u}{\partial \sigma} - \frac{\partial \left(\frac{u^2 + v^2}{2} \right)}{\partial x}$$

$$\frac{\partial v}{\partial t} = -\eta \frac{u}{v} - \frac{\partial \Phi}{\partial y} - c_p \theta \frac{\partial \pi}{\partial y} - z \frac{\partial v}{\partial \sigma} - \frac{\partial \left(\frac{u^2 + v^2}{2} \right)}{\partial y}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \omega \left(\frac{\partial T}{\partial p} - \frac{RT}{pc_p} \right) = \frac{J}{c_p}$$

To these basic equations must be added more physics...

- The energy change due to water changing from vapor to liquid and to ice
- Radiation transport equations, describing how photons from the sun and Earth diffuse through the atmosphere and get absorbed and re-emitted. And also how collisional excitation of the molecules can carry thermal energy into radiation energy and vice versa
- Transport of energy to/from the ocean surface *via* waves and evaporation
- Aerosols production (e.g. salt crystals, desert dust, volcanoes) and effect on radiation transport and ability to store, re-radiate heat
- Clouds and how they alter radiation transport, emissivity, including the differing cloud types and altitudes
- Production of human-made aerosols and gases, and changes to the albedo of the Earth due to human activities (which take little heed to the laws of physics!)
- Release of methane from permafrost as it melts (not included in modern models yet)
- Interactions with the biosphere – CO₂ into/out of the ocean, plants,
- etc.

Calculus vs. Finite Difference Codes

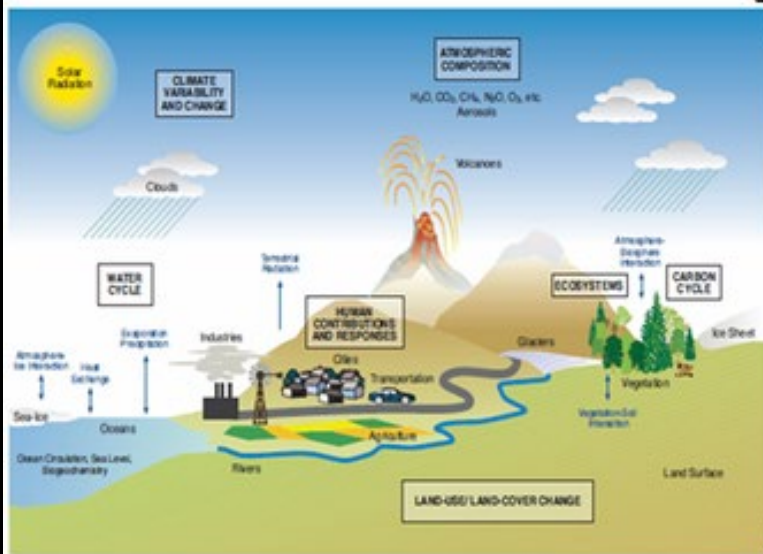
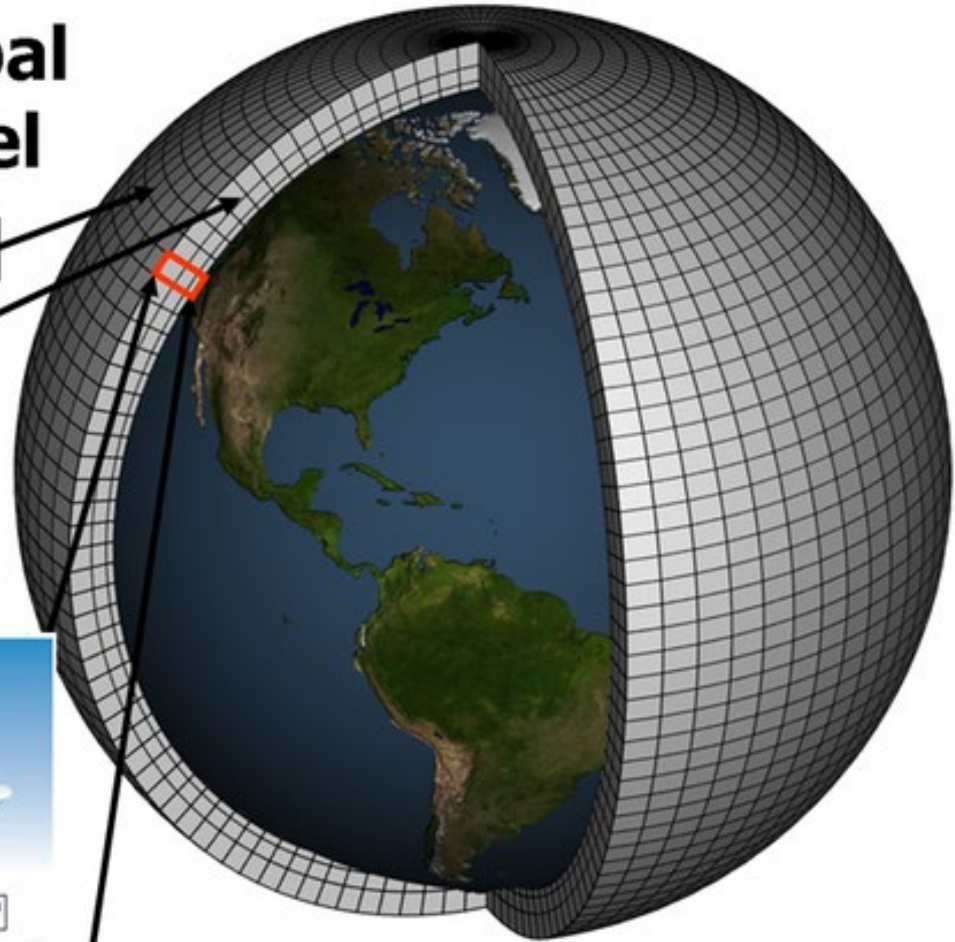
- These are differential equations – In fact, **Almost ALL of Nature's laws are Differential Equations!**
- The boundary conditions and initial conditions for these equations are far too irregular to allow closed-form solutions like you saw in elementary math classes.
- The only realistic solutions are by computer...
- Convert the differentials into differences (convert calculus into arithmetic – computers understand arithmetic!)
- That means divide up the Earth system into little boxes or elements, knowing that for any given quantity (temp, pressure, mass, whatever) you will only have a single measurement for that entire element. You lose the ability to know the varying details within the element

Computer Models are Finite-Difference Versions of the Differential Equations Governing The Physics

Schematic for Global Atmospheric Model

Horizontal Grid (Latitude-Longitude)

Vertical Grid (Height or Pressure)



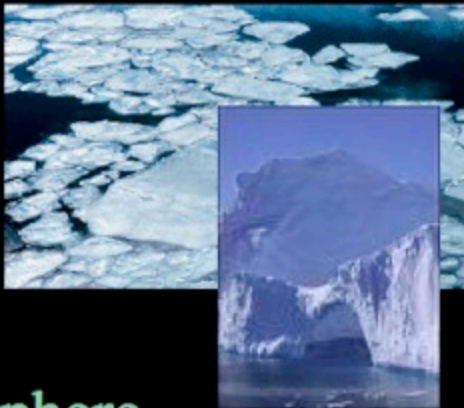
GCM: Global Climate Model



Atmosphere



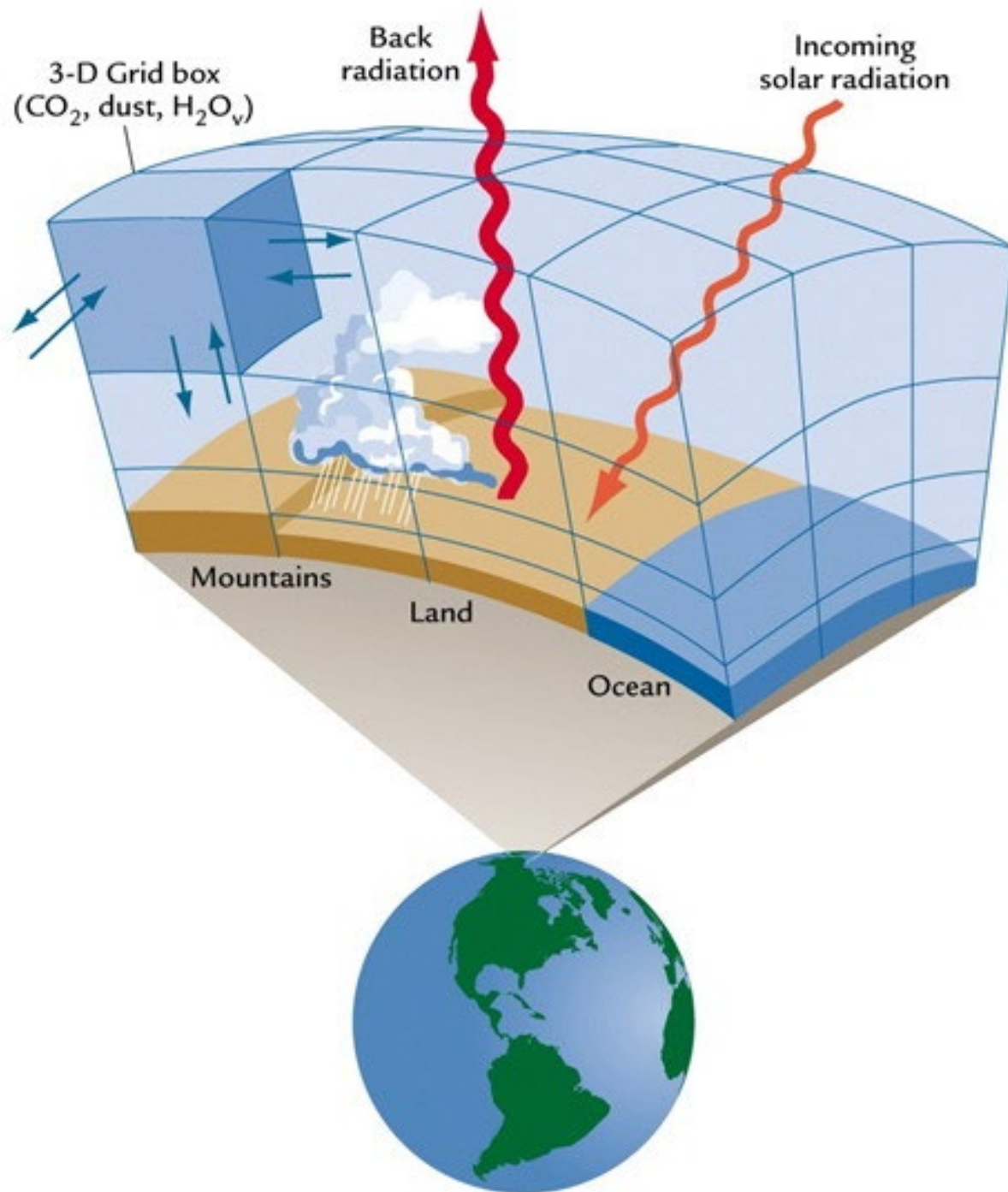
Oceans

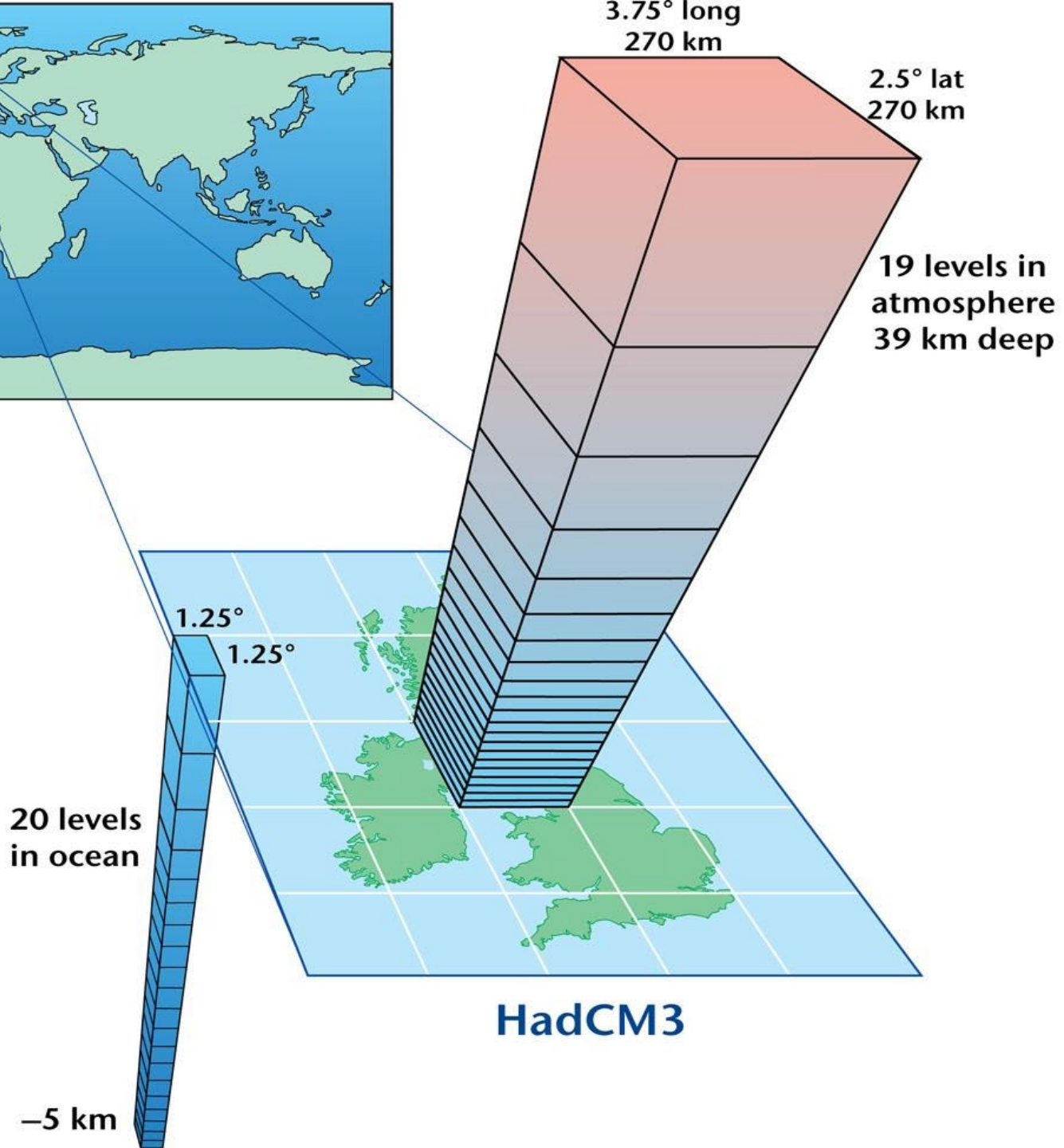
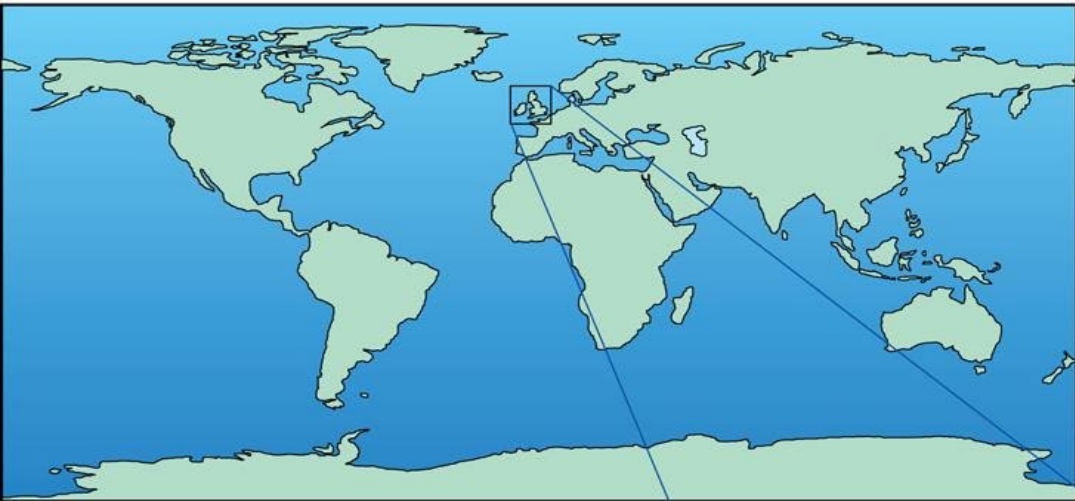


Cryosphere



Vegetation





Climate Models...

- ... are systems of differential equations based on the fundamental laws of physics, fluid dynamics, and chemistry.
- To “run” a model, scientists divide the planet into cells in a 3-dimensional grid, apply the basic equations in difference form in which they can yield answers, submit them to a computer and run it, and evaluate the results.
- Atmospheric models calculate winds, heat transfer, radiation, humidity, vorticity production and transport, and interaction with surface water for each grid cell and evaluate interactions with neighboring grid cells.

Fundamental Physical Quantities & Equations

At every grid cell GCMs calculate:

- ▶ Temperature (T)
- ▶ Pressure (P)
- ▶ Winds (U, V)
- ▶ Humidity (Q)

- Conservation of momentum

$$\frac{\partial \vec{V}}{\partial t} = -(\vec{V} \cdot \nabla) \vec{V} - \frac{1}{\rho} \nabla p - \vec{g} - 2\vec{\Omega} \times \vec{V} + \nabla \cdot (k_m \nabla \vec{V}) - \vec{F}_d$$

- Conservation of energy

$$\rho c_v \frac{\partial T}{\partial t} = -\rho c_v (\vec{V} \cdot \nabla) T - \nabla \cdot \vec{R} + \nabla \cdot (k_T \nabla T) + C + S$$

- Conservation of mass

$$\frac{\partial \rho}{\partial t} = -(\vec{V} \cdot \nabla) \rho - \rho (\nabla \cdot \vec{V})$$

- Conservation of H_2O (vapor, liquid, solid)

$$\frac{\partial q}{\partial t} = -(\vec{V} \cdot \nabla) q + \nabla \cdot (k_q \nabla q) + S_q + E$$

- Equation of state

$$p = \rho R_d T$$

Another Look....

Conservation of momentum, energy, mass and moisture:

$$\frac{\partial \vec{V}}{\partial t} = -(\vec{V} \cdot \nabla) \vec{V} - \frac{1}{\rho} \nabla p - \vec{g} - 2\vec{\Omega} \times \vec{V} + \nabla \cdot (k_{\omega} \nabla \vec{V}) - \vec{F}_d$$

$$\rho c_p \frac{\partial T}{\partial t} = -\rho c_p (\vec{V} \cdot \nabla) T - \nabla \cdot \vec{R} + \nabla \cdot (k_{\tau} \nabla T) + C + S$$

$$\frac{\partial \rho}{\partial t} = -(\vec{V} \cdot \nabla) \rho - \rho (\nabla \cdot \vec{V})$$

$$\frac{\partial q}{\partial t} = -(\vec{V} \cdot \nabla) q + \nabla \cdot (k_q \nabla q) + S_q + E$$

Equation of state:

$$p = \rho R_d T$$

V = velocity

T = temperature

p = pressure

ρ = density

q = specific humidity

g = gravity

Ω = rotation of Earth

F_d = drag force of Earth

R = radiation vector

C = conductive heating

c_p = heat capacity, constant p

E = evaporation

S = latent heating

S_q = phase change source

k = diffusion coefficients

R_d = dry air gas constant

Short Videos Worth Seeing

- [“Climate Modelling 101”](#) from the National Academy of Sciences...
- ...Includes this [YouTube 4 minute primer on constructing a climate model](#)
- An math-wonky 9 min YouTube on [Climate Modelling](#), from the University of Victoria - Canada
- [Grid Resolution in Climate Models](#) (9 min) (skip forward to time 1:35) from NOAA’s Geophysical Fluid Dynamics Lab, Princeton. (Slow, but the simulation video of ocean flows is impressive and interesting)

How Do We Streamline Models So As to be Do-able for a Finely Gridded Globe or Long Time Scales?

- Most difficult to model is clouds, and to a bit lesser extent aerosols. Too complex on too many size scales to be able to calculate the creation of all clouds over the whole Earth! While we know the equations, there are no computers big enough or fast enough to evaluate the bazillions of calculations. So...
- Embed simpler, quicker 1- and 2-dimensional cloud modelling into the larger 3-dimensional grid models (Super-parameterization). 3D modelling is the most computer-intensive, slow and expensive!
- Parameterize behavior of a system using observational data, so that intractable calculations can be avoided. Parameterize either as a fitting formula, or look-up table.
- Model subsystems using parameters determined from real data, and embed these into the larger 3-D code
- Assign fractionals; e.g. fractional cloud cover in a cell, rather than just 'clear' or 'cloudy'.

More tricks....

- use Fast Fourier Transforms to convert difficult integrals into arithmetic calculations, then invert at the tail end.
- Use adaptive grid sizes**; *e.g.* let grid size be large over uniform areas like many in the oceans, and small over complex areas like mountain ranges and coastlines.
- **Use adaptive time steps**; long when forcings are small and conditions change slowly, and short when forcings are large and change is rapid

But aren't physical systems chaotic - The "Butterfly Effect"? Doesn't that make climate fundamentally indeterminate?

- **No.** And those who should know better mis-use it often to advance their anti-science agendas!
- **Weather, however, IS somewhat chaotic.** You can't tell the temperature at Cabrillo at 4:14pm on Dec 18, 2041. Beyond a week, weather forecasts are unreliable because of chaos
- But **climate is *time-averaged* weather**, and very slow change in the fundamental forcings guarantee that climate has only a small amount of chaos, within tight bounds, in the same way that seen from a distance, a mountain stream looks very predictable, but up close over tiny time scales it looks chaotic and unpredictable.

Chaos is Different than Tipping Points

- Climate DOES have “tipping points” which, when crossed, send climate into a new regime which may be very different. Tipping points are different than chaos.
- Climate change is like a dog on a fairly tight leash as you go for a walk. The “walk” direction is the changes predicted from climate forcings. The dog’s darting this way and that is like the weather. Chaos, is a frisky puppy on no leash. He could go ANYwhere!
- A [good discussion of chaos in climate](#), in a piece honoring the pioneer of chaos theory in weather and climate.

Well that's just reasoning – **how can you really tell?**

- Straightforward to answer that! You re-run your climate model and make random (but reasonable) changes to your initial conditions (the initial temperatures, atmospheric pressures, humidities, etc.) which start your GCM and then run it again, over and over, and see how the end state changes as you vary those initial conditions.
- If they change a lot – you've got chaos problems!
- Do this for real-world “postdictions” especially. The changes in the end result reflect how sensitive your model is to chaos.

Result? Chaotic effects are seen to be very small compared to the climate changes we seek to understand.

- In the same way that photographs of a churning mountain stream taken from 100 feet away at 20 different times all within the hour (climate), will look very very similar, while the same stream photographed at the same time but from only a few inches away, will look rather different in churning details (weather).
- But suppose you're "forcing" your stream with slowly rising water volume. A point may come where it over-flows a bank and makes an entirely new channel, then it will look very different!
- This is not chaos, it's a "**tipping point**"

Climate Model Tipping Points

- Tipping points are the exception. Positive feedbacks can amplify a forcing to the point that it takes you to a new regime.
- For human purposes, it's extremely important to know exactly when a climate tipping point will occur – how far you can force the climate system before it transitions to a new regime?
- Unfortunately, the exact point of the “tip” is hard to calculate reliably, so we try to use paleo climate to help inform us.
- What we're doing today is playing “chicken” with these tipping points

Tipping Point Examples

- **Runaway Greenhouse**, where it's so hot that evaporated water rises sufficiently to the stratosphere that it can be UV- dissociated and permanently lost to space, causing a run-away loss of water
- **Methane loss from subsea methane hydrates**; if quick enough and large enough, can add enough extra heating to cause the thaw to accelerate, leading to fairly rapid transition to methane release that is faster than the decade-long half-life of methane in the atmosphere, and thus leading to vastly stronger heating, taking climate potentially to a very different place.

And One that We Will See is Important Later in Our Real Climate...

- **Tundra thaw**, whereby the Arctic becomes a net source of carbon instead of the sink (via enhanced plant growth) of today, leading to rising greenhouse sufficient to ensure rapid thaw of all permafrost and release of its carbon. Paleo data indicates this happens at global average temperatures of +1.5C above pre-industrial, very close to today's.

So – Again: How to check for Chaos affecting results?

- You run your model many times, each starting with different but realistic initial conditions. If chaos is minimal, results should be similar in different runs. If the results **aren't** close to the same, chaos may be affecting the results.
- Real GCM experience shows **chaos is not a significant problem.**
- Tipping points. We try hard to predict where they will happen

How Climate Models Are Tested for Accuracy: Internal Checks

- As of 2011, there were 40 different full-scale GCM climate models being regularly improved and run, worldwide
- Internal Checks (*i.e.*, is the model self-consistent?)...
- 1. **Does it include the relevant physics?** If including additional physics effects do not change results, the new physics is likely not important to real-world climate
- 2. **Do the predictions change significantly on a large size scale or global level when the grid resolution is increased?** If yes, then numerical resolution uncertainties are still significant. If not, then you may have indeed “converged” onto the actual solution and further resolution improvement may not be cost-effective
- 3. **Do the basic conservation laws of physics remain satisfied?** *i.e.* is energy conserved? Is angular momentum conserved? Is mass conserved? You might think these are explicitly made to be true, but in a finite element and finite time-stepped algorithm, then numerical effects can cause non-conservation as the time steps continue, if one is not careful.

Modelling Improvements...

- Always, more computer power will come. Already, we're using 100,000 trillion floating point operations to simulate one century's worth of climate (in 2011). Computer power has increased by a factor of 10 million in the past ~30 years
- Ice sheets non-linear breakup and glacier base lubrication. These can affect climate significantly! ...Being added as we speak, but **not included in models discussed in IPCC AR4 and AR5**
- Adding additional chemical processes (there's many, after all. Are these going to be significant? Evidence says probably not)

- **Interactive biology**; example, we may need sea animal-induced turbulence to provide enough mixing in mid layers of the ocean to account for observations. Seasonal greening done only simplistically so far. Changes in species, although that's hard to guess and simulate, and fortunately probably not climate-significant
- Predict human behavior as a dependent variable in the models, since clearly we are a major factor in climate now and going forward (yeah – I personally would give up on this one. Only can do if **human rational predictability is assumed – is that realistic? Hasn't worked out that way**).
- Add in more identified positive feedbacks: methane from thawing permafrost, darkening polar snow, moulins and base water in ice caps, mixing out of marine clouds
- In general, try hard to improve **cloud and cloud/aerosol modelling. This remains the hardest part of climate modelling, because important physics happens even on small scales we can't access from global models**

How Climate Models Tested for Accuracy: External Checks

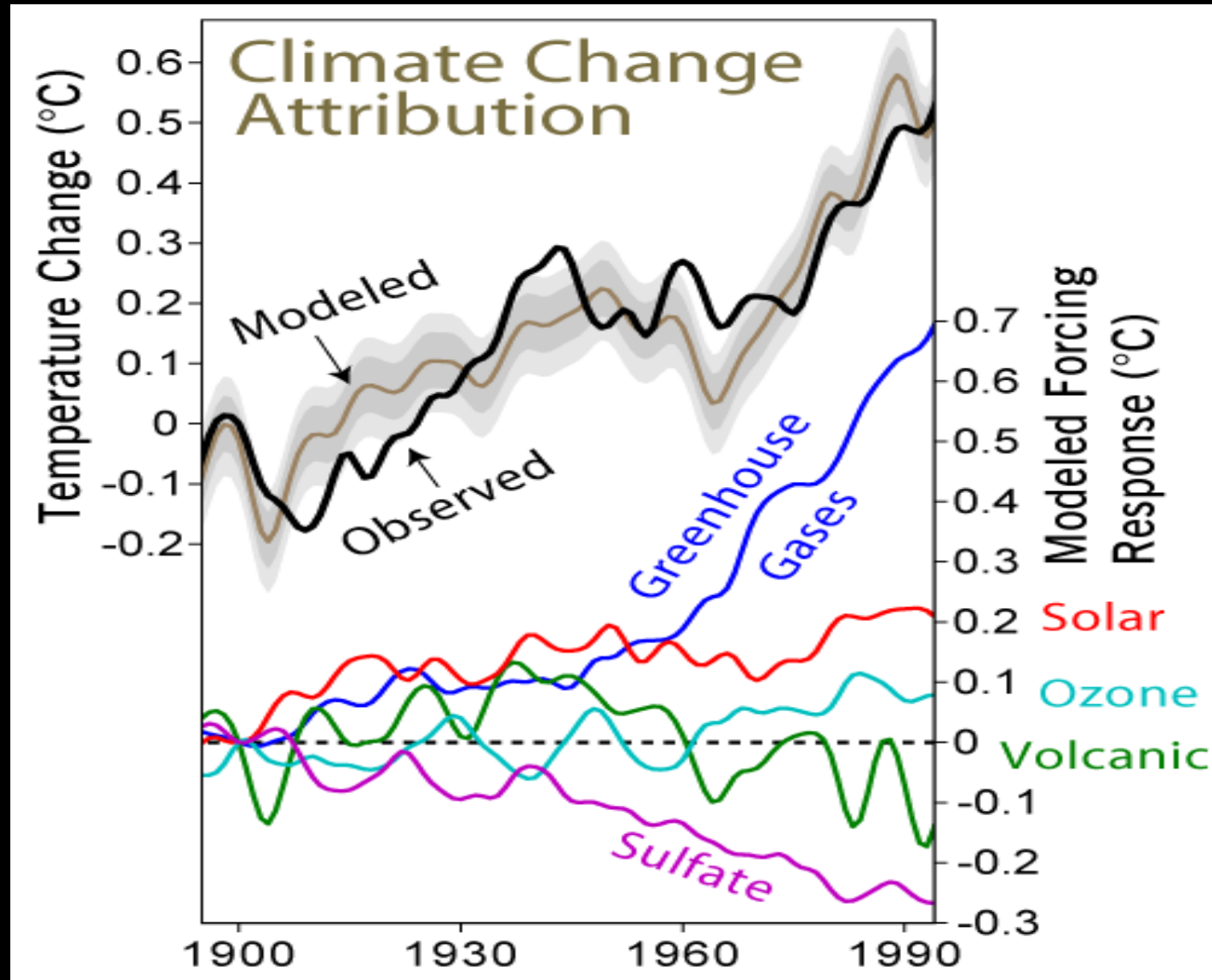
- **Can you run your model or important subroutines on a simple system for which real-world answers are known, and does it reproduce those results? Example – laboratory set ups of fluid systems, heat transport, etc.**
- **Can you run your model on a complex system and still reproduce the real-world data fairly well, over the time and size scales the climate model is designed to be relevant?**

- **Weather Forecasts:** A state of the art climate model ought to be able to do weather forecasting. After all, the time step used to march the model forward within the computer is about 1 minute. Since future weather becomes past weather in just a week or so, and since there are a huge number of different weather stations around the world, you can make a decent statistical base of how well your climate model does in forecasting weather, without waiting for long term climate to happen. It does require you to initialize your run with known actual real-world data at an instant in time, which can be challenging.
- Climate models usually initialize with reasonable average conditions for each cell.
- **Modern climate models have done well in both internal and external checks.**
- **...Provided all the relevant physics is included! This is not yet the case. e.g. methane release, glacial base lubrication, rafted ice melt, etc.**
- Today's GCMs produce systematic errors over time scales of a century which are comparable to the error in weather forecasts over several days, which is doing pretty well.

GCM's – A Good Record of General Accuracy

- As far back as 1975, the first coupled ocean – atmosphere general circulation (GCM) model was developed and run to try and determine the effect of doubling the level of CO₂ in our atmosphere, by [Manabe and Wetherald \(1975\)](#)
- They predicted: (1) Most warming at the poles, especially the Arctic, (2) Warming of the troposphere, (3) Cooling of the stratosphere, (4) More rain overall, (5) Higher absolute humidity
- All of these predictions have come true and are intensifying as we speak. The Arctic has warmed about twice as much as the global average pretty much as predicted.
- Of course, you have to include the mounting non-linear physics of ice sheet instability, cloud changes, methane production, etc, or these models won't be reliable for predicting the FUTURE, which is what we are most interested in!

How Well Do Climate Models Work? Pretty Good when tipping points aren't crossed (alas, they're now being crossed).



However...

- Climate change so far has been mild, and feedbacks have been not too strong. Going forward this is not likely the case... Arctic Ocean will be ice-free in the summer soon.
- Glacial terminus grounding, wildfires continue to darken polar ice, carbon release from the thawing Arctic is becoming significant, permafrost taliks, glacier base lubrication from moulin flows, Equilibrium Climate Sensitivity (ECS) dependence on climate state, etc... these and other factors are not yet in current climate models
- So **post**-dictions have been good, but this does not mean **predictions** for decades into the future are reliable. Observations strongly suggest predictions from models are still too conservative about future climate
- The next IPCC AR6 in 2022 we expect will be significantly more dire than those of the past.

Key Points: K34 - Climate Modelling

- Climate governed by Laws of Physics: Coupled non-linear partial differential equations with complex boundary and initial conditions
- Cannot be solved in closed form; only by computer. Computer solving – re-write DiffEq's as difference equations (which are always solve-able), set up a grid of finite elements (“cells”).
- Set initial conditions from real-world data, decide time step, and march forward in time as far as your computer grant \$\$ will take you!
- Typical **state of the art time step: 1 minute, grid cell size: 100 km**
- Well-modelled: Solar input, ocean/air, volcanic aerosols, radiation physics, and more physics...
- **Not as well: clouds and cloud/aerosol interaction** – reason is that clouds exist on spatial scales far smaller than can be modelled in a global circulation model. **Cloud dynamics are the most difficult ingredient to model well**
- **So: Coping Strategies** - embed 1,2-D models into your 3-D GCM. Parameterize real-world data into a “lookup table” or fitting formula for deciding e.g. cloud cover vs altitude for given humidity, thermal profile, forcings. Assign fractional cloud cover to a given cell, rather than only “clear” or “cloudy”.
- Feedbacks: positive= amplify initiating direction. Negative= inhibit initiating direction
- Feedbacks cannot change the sign of the initiating direction because zero change means zero feedback. (So feedbacks are not going to save us from AGW).
- Chaos – weather is chaotic, but only within limits. Climate (=time averaged weather) is much less so. **Chaos Check:** is GCM stable against reasonable random variance in initial conditions?
- **Internal checks:** solutions stable against change in grid size? Chaos Satisfy conservation laws? Is the time step small compared to the change time scale of calculated quantities? Is the spatial scale small compared to how physical quantities vary across space? Is all important physics included?
- **External checks:** Does model predict weather on short time scales? Does it reproduce lab results on real-world simpler systems? Can it reproduce past (recent and well measured) global climate using “post-dictions” from realistic initial conditions.
- **How do Modern GCM Models Do?** Pretty good so far on all measures. Can trim errors further if we can improve cloud and aerosol modelling, but these are relatively minor compared to the large anthropogenic forcings that we most want to understand and forecast with. GCM's can predict ahead for 100 years with accuracy comparable to weather forecasts ahead by a few days. Much better than a random guess.
- But, they don't yet include some important positive feedbacks identified in the past 15 years, like Arctic carbon release, ice sheet breakup dynamics, glacier base lubrication from water into moulins, darkening snow from wildfire ash, thawing tundra, mixing out of low level clouds
- Tipping points are hard to calculate with the precision we want, when climate transitions to a new state rather than a small difference from the existing state.