Chapter 4: Energy, Motion, Gravity

Enter Isaac Newton, who pretty much gave birth to classical physics

Chapter 4 – Key Points

- Know all of Kepler's Laws well
- Acceleration proportional to force, inverse to mass (Newton's 2nd law)
- ALL forces are between objects, each feels the same force; equal and opposite direction
- Know why we see retrograde motion in planets, how explained by Copernicus
- Venus shows all phases, supports Copernicus, disproves Ptolemy Earth-centered model
- Know how gravity behaves in a spherically distributed mass, inside and outside
- The tidal force – because gravity force drops with increasing distance. It's a stretching force, larger for larger objects because of the larger <u>difference</u> in gravity side to side. Tidal friction drags moon forward, at the expense of Earth's rotation
- Even circles are ellipses; both foci at the same place
- Know the concept of Angular Momentum, feel how much "angular oomph" there is in a system; that is it's angular momentum. Since gravity acts only along the radial direction, it can't help nor hurt the angular momentum, hence it is "conserved".
- Angular momentum remains constant in a system not acted upon by outside forces: Conservation of Angular Momentum

More Trouble-making from Galileo: His Experiments

- Aristotle taught “gravity – the tendency of heavy things to fall”, and: heavier objects will fall faster than lighter objects. The Church adopted this as gospel
- Is that right? Pretty easy to discover by yourself by just dropping things…

Galileo also did experiments in the motions of objects

- Galileo dropped objects (even from the Tower of Pisa, according to legend anyway) to see how they fell…
- --The rate of acceleration of falling objects is a **constant**. 32 ft/sec per second, or about 10 meters/sec per second, for as long as it falls.
- -- Objects fall at the same rate, regardless of their mass, temperature, color, composition…
- -- Measured the period of the swing of pendulums, found it was (for amplitudes not too large) the same regardless of the amplitude of the swing. (in truth, the period is only extremely close to constant for small amplitudes).

Without air friction, feathers and hammers fall at the same rate…

- YouTube Feather/Hammer on the Moon
- Indeed, you see they fell at the same rate!
- Galileo’s experiments were the perfect set up to allow Newton to identify the foundations of the branch of physics we call Mechanics: The behavior of objects under the influence of forces

Newton’s 3 Laws of Motion

- These are more general than gravity.
- They’re the basis of the branch of physics called…
- ***Mechanics*** *– how objects move when under the influence of* ***forces***

Newton's 1st Law

- **The Law of Inertia**
- ***An object will remain in it's same***
- ***state of motion unless acted on by a force***
- A revolutionary idea at the time, as Aristotle (the Church's chosen physics authority) taught "The natural state of motion of an object is to be at rest" (and therefore requiring no explanation), and here, Newton discovered that moving things will stay moving, and slowing down requires a force to be acting.
- Aristotle apparently didn't grasp the idea of friction as a force
- The 1st law is really a special case of a more general law…

Newton's 2nd Law

- ***The acceleration an object experiences is directly proportional to the <u>force</u> acting on it, and inversely proportional to the <u>mass</u> of the object***
- **Acceleration = Force/Mass**
- In plain English – ***heavier things are harder to push up to speed***, and the harder you push, the faster it'll accelerate. Your intuition should serve you well here!

Newton's 3rd Law

- Forces between objects are always felt mutually; equal and opposite in direction
- Often called the ***Law of Equal and Opposite Reactions***

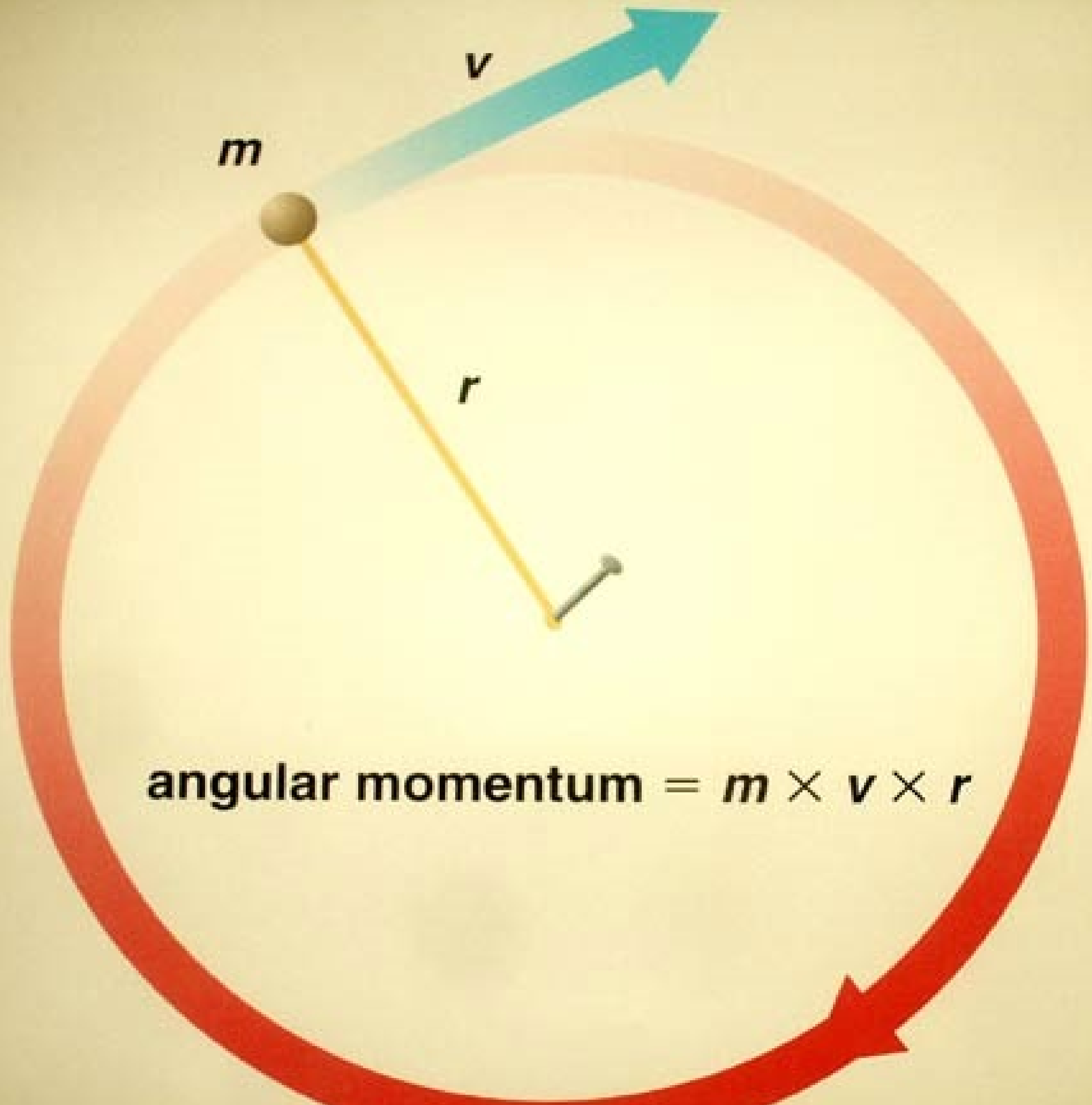
- In plain English… when you push or pull on something, it'll pull or push back, equally

These Laws of Motion, plus Galileo's Observations, Allowed Newton to Infer the Law of Gravity

- But before we do this, let's review how Kepler's 2nd Law relates to Angular Momentum.

Kepler's 2nd Law and Conservation of Angular Momentum

- Notice that how a planet speeds up as it gets closer to the sun, is exactly such as to keep the amount of "angular oomph" the same, anywhere in the orbit.


v
m
r
angular momentum = m × v × r

The Meaning of Angular Momentum

- Imagine something moving around an orbit, or maybe around its own axis of rotation.
- Now imagine how much effort you'd have to do to STOP that angular motion.
- The amount of that effort is a good feel for its ***Angular Momentum.***

Conservation of Angular Momentum

- In a closed system of bodies (*i.e.* not influenced by any outside objects), the total angular momentum of all objects is “conserved” (*i.e.* it doesn’t change with time).
- It’s a consequence of Newton’s 3rd Law.
- Kepler’s 2nd law is essentially the application of the The Law of Conservation of Angular Momentum to planetary orbits, together with the fact that (which we’ll learn about later, and is indeed a force), <u>gravity acts only along the line between the objects – so it can’t add “sideways” impetus (angular momentum) to what’s already there to begin with.</u>

- A weblink animation showing an object in a Keplerian Elliptical orbit which can be varied

Newton Used his Laws of Motion, Galileo's Observations, and the motion of the moon to Make a Good Guess at the Law of Gravity

- Let's follow his reasoning. Sit under the apple tree with Sir Isaac and ponder… while I work things out on the white board…

When you drop two different masses, they fall at the same rate

- But that acceleration rate must be proportional to the FORCE applied to it, according to Newton's 2^{nd} Law of Motion.
- And, the acceleration is INVERSE to the mass of the object being accelerated (his 3^{rd} Law).
- So the observation that different masses on Earth fall at the same rate, tells us…

First – that Gravity is a FORCE

- Because it's clearly imparting an ACCELERATION to what's falling!
- And second, because that acceleration is the same no matter the mass, that means that the FORCE of gravity must be proportional to the MASS of the object you're watching being gravitated!

So we have the Beginnings of the Force Law of Gravity…

$$F = m_o$$

Next we ask – What else may make a difference and so must enter this equation?

- Galileo showed that temperature, color, chemical composition, shape, etc made no difference. They don’t enter the Law of Gravity
- But things are falling to **Earth**. And they don’t fall towards a smaller “planet” like that rock over there.

Even if we deem and declare this to be an official PLANET. Still, it doesn't show any measurable attractive power.

You All Can Guess Why… it has such a tiny mass compared to the Earth.

- Let's take that guess and run with it…
- The Earth is attracting the rock, but is the rock attracting the Earth? Yes! It has to – by Newton's 3rd Law of Motion
- Equal and Opposite Reactions says YES.
- And therefore the Law of Gravity must depend symmetrically with the other mass.

So Our Law of Gravity Has Made Progress, and Now Looks Like This…

$$F = m_o m_{earth}$$

The Last Thing Galileo Couldn't Test as a Variable: Separation

- Because everything he experimented was about 4,000 miles from the center of the Earth.
- Even going to the top of nearby Mt Blanc in the Alps would only give him an extra 3 miles. Less than 1/1,000 of an additional distance, and likely far too small to detect differences given the crude instruments of the day

But Newton Realized that if the Universe was Really Governed by NATURAL Laws

- …then the attraction of rocks to the Earth probably extended out to the Moon, and explains why the Moon follows its accelerated motion called…
- Orbiting!
- Orbiting, after all, is continuous falling; together with constant sideways momentum

Newton Knew How Far Away the Moon Was

- It was 60 Earth radii away.
- He also knew how long it took to orbit the Earth – 27.3 days
- From that you can calculate its ACCELERATION.
- He found the moon was accelerating towards Earth by 1/3600th of the acceleration of rocks at the surface of the Earth

Hmmmm. 60 squared is 3600

- And he sees that the moon is 60x farther away and falling $1/60^2$ as much.
- So we have new progress. The Law of Gravity now is seen to be…

- $F = m_o \, m_{earth}/r^2$

Newton was Almost Done…

- But there's one more thing. The dimensions in this equation are not right yet. The left side is "force" and the right side is mass2 /distance2.
- Clearly we need a dimensional constant. Christen it "Newton's Universal Constant of Gravity = G"
- Think of G as the number that contains how strong gravity is as a force.

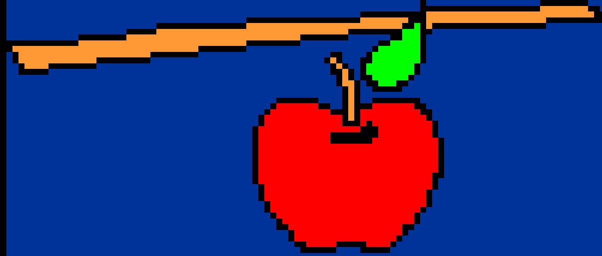
Gravity is actually a fantastically weak force. By far the weakest in Nature

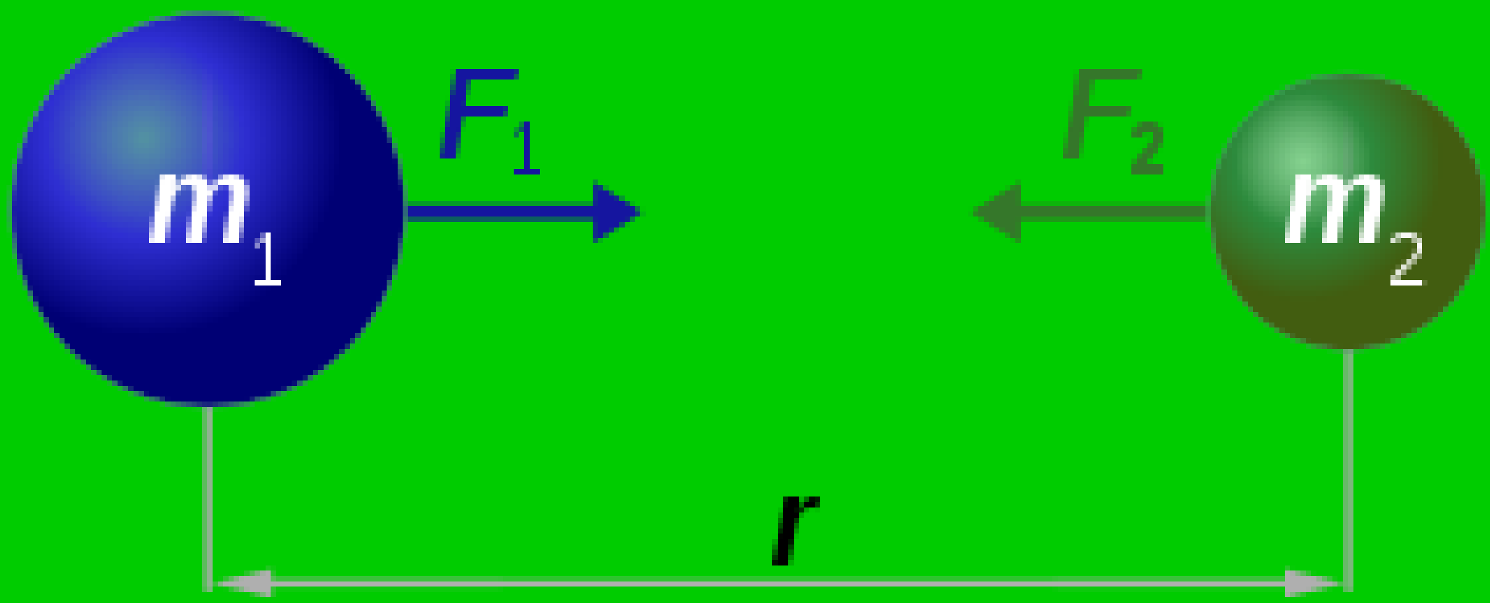
- You could have guessed that, because it takes the entire EARTH pulling on an average rock to give a weight of one little pound.
- So G is a very tiny number
- $G = 6.68 \times 10^{-8}$ $cm^3/gram\text{-}second^2$

Our Final Expression for Newton's Law of Gravity

- $F = Gm_1m_2/r^2$

- Now what's so great about Newton, was his first thoughts after arriving here, was that there was a great way to TEST this.
- It's a short step from realizing the moon feels gravity, to the planets feel gravity from the sun.





$$F_1 = F_2 = G\,\frac{m_1 \times m_2}{r^2}$$

Confirmation of the Law of Gravity by verifying it predicts Kepler's 3 Laws

- Newton realized that if gravity held you and me to the earth, and held the moon in orbit, it was a short jump to infer the planets were held in orbit by gravity from the sun. So…
- Kepler's Laws (which still were unexplained) must be derive-able by pure reasoning from Gravity and the laws of motion.
- Not easy – had to discover the rules obeyed in the branch of mathematics called "calculus" first. That was his summer vacation of 1666.

Kepler's 3rd Law as Derived by Newton

- Kepler's 3rd Law is $P^2=ka^3$
- Newton's derivation of this law looked like this…
- $P^2 = 4\pi^2a^3/G(m_1+m_2)$
- Did somebody goof? Why don't they look the same???

No Goof…

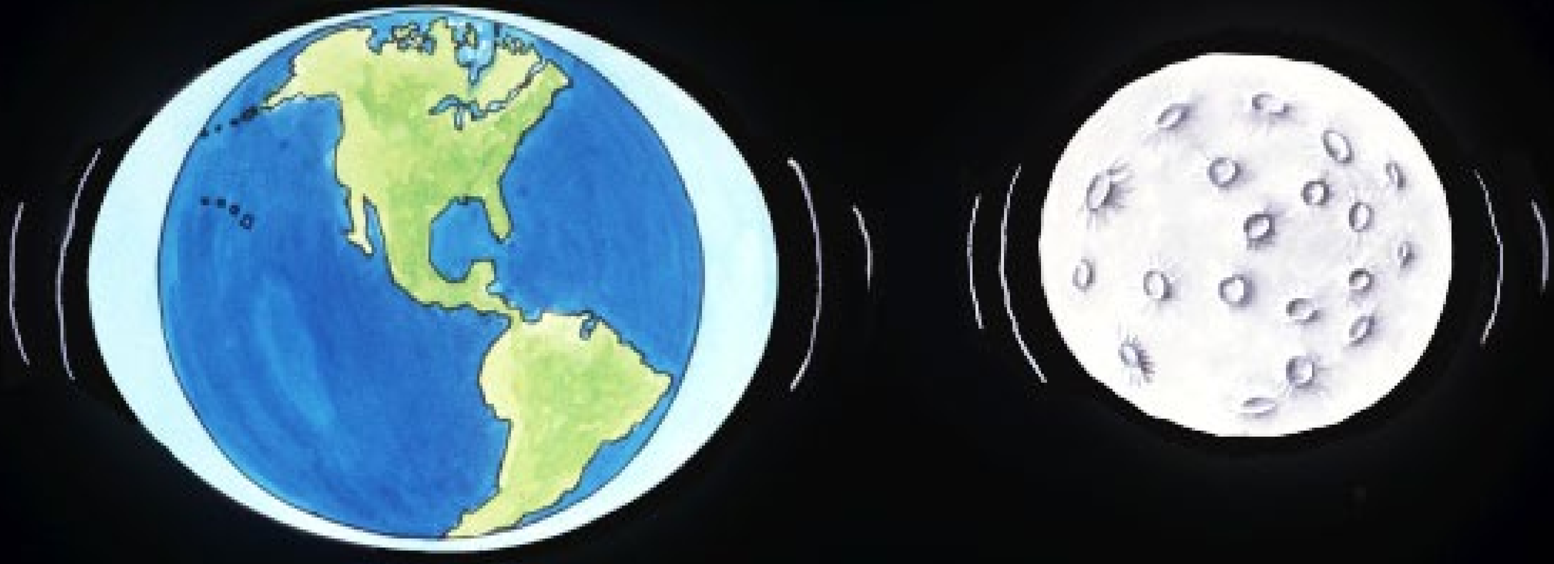
- Kepler version is a simple quantitative fit to what he saw in Brahe’s data, it was an induction, as accurate as was Brahe’s (pre-telescope) data.
- Newton’s version is an exact deduction given his law of gravity. His form looks different because Kepler’s “k” blossomed into a whole combination of other physical constants. Beautiful!

Note that gravitational force is stronger when things are closer

- A direct consequence of this is the phenomenon of **tides**.
- Tides are far more general than just water moving up and down on the earth.
- Tidal Force accounts for much of why the solar system, stars, and galaxies are the way they are – tides are IMPORTANT!

The Tidal Force

- Not really a new force; it's an aspect of gravity.
- Gravity is stronger when closer. So, the near side of an object will feel more attraction than the far side, causing a stretching force.
- What will this gradient in gravity do to the Earth's shape?...

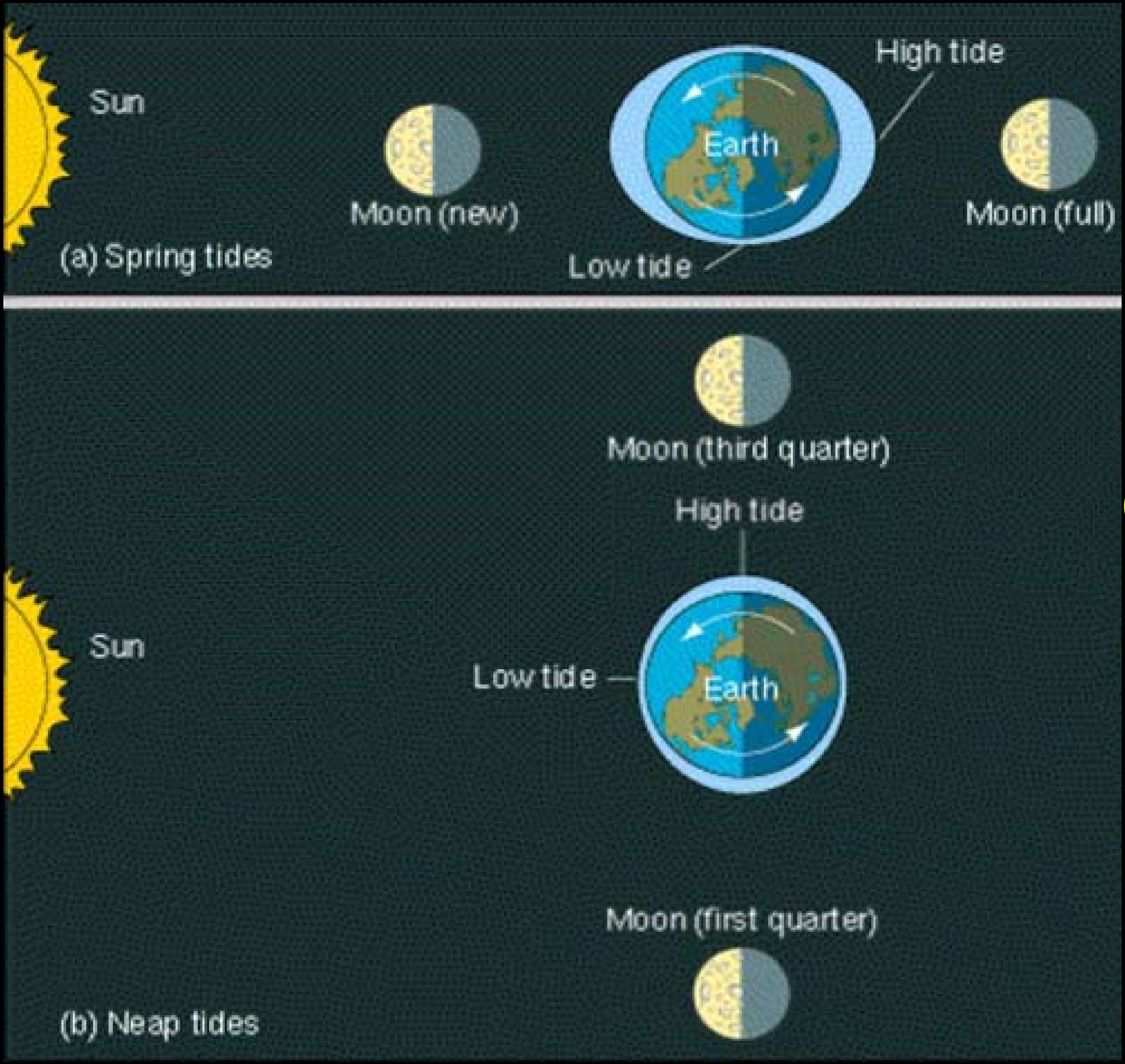


Now Bring in the Sun to this discussion…

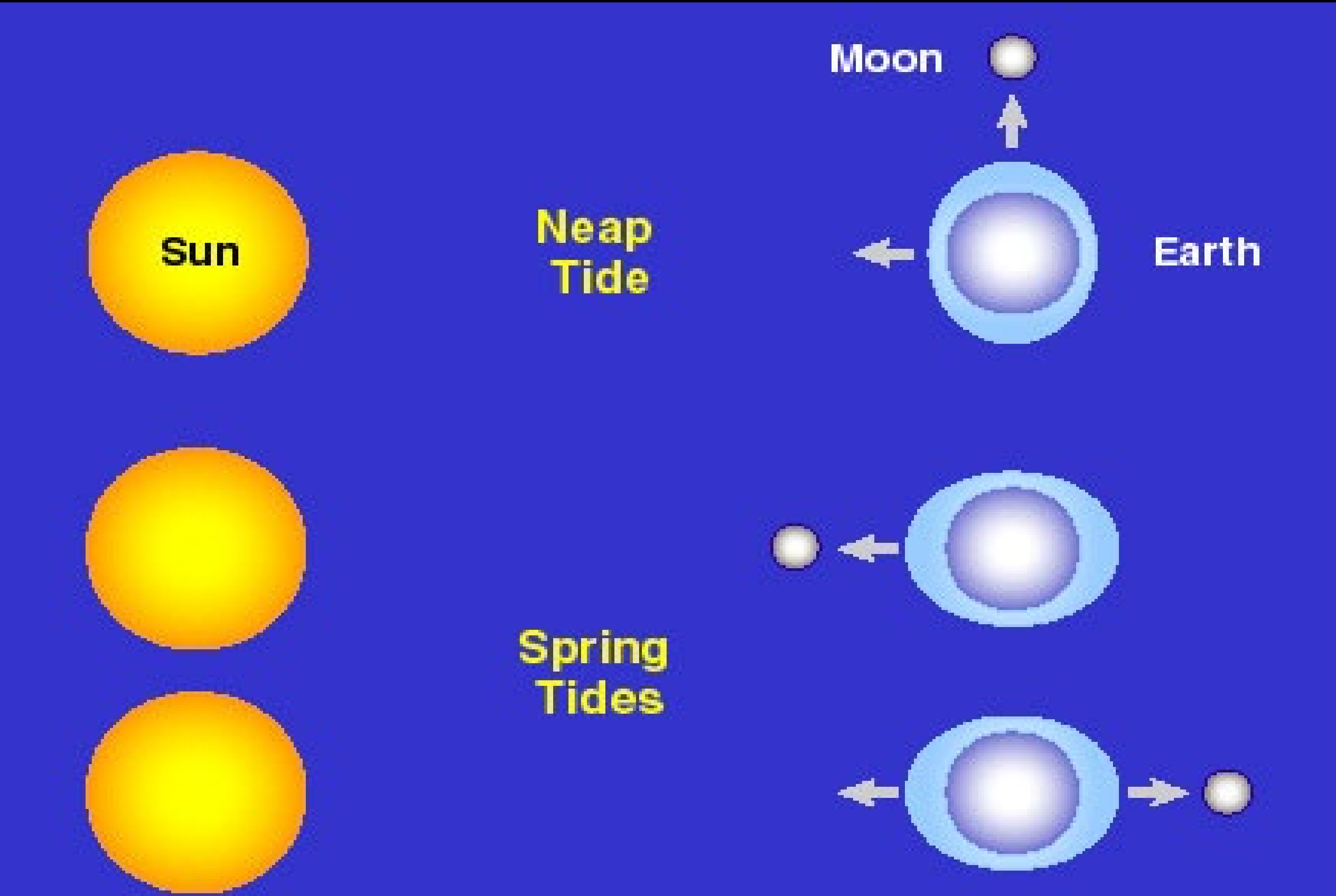
- What do you guess – will the sun raise BIGGER or SMALLER ocean tides than the moon?
- Can you give reasonable arguments?

Let’s compare the tidal stretching due to the Sun vs. the Moon

- The sun is 400 times larger in diameter than the moon.
- But the moon is 400 times closer
- If the density of the sun and moon were the same, they’d have the same tidal stretching effect on the Earth.
- But the moon is denser; tidal stretching from the sun is only 46% that of the moon.
- Because the sun is 400 times further away

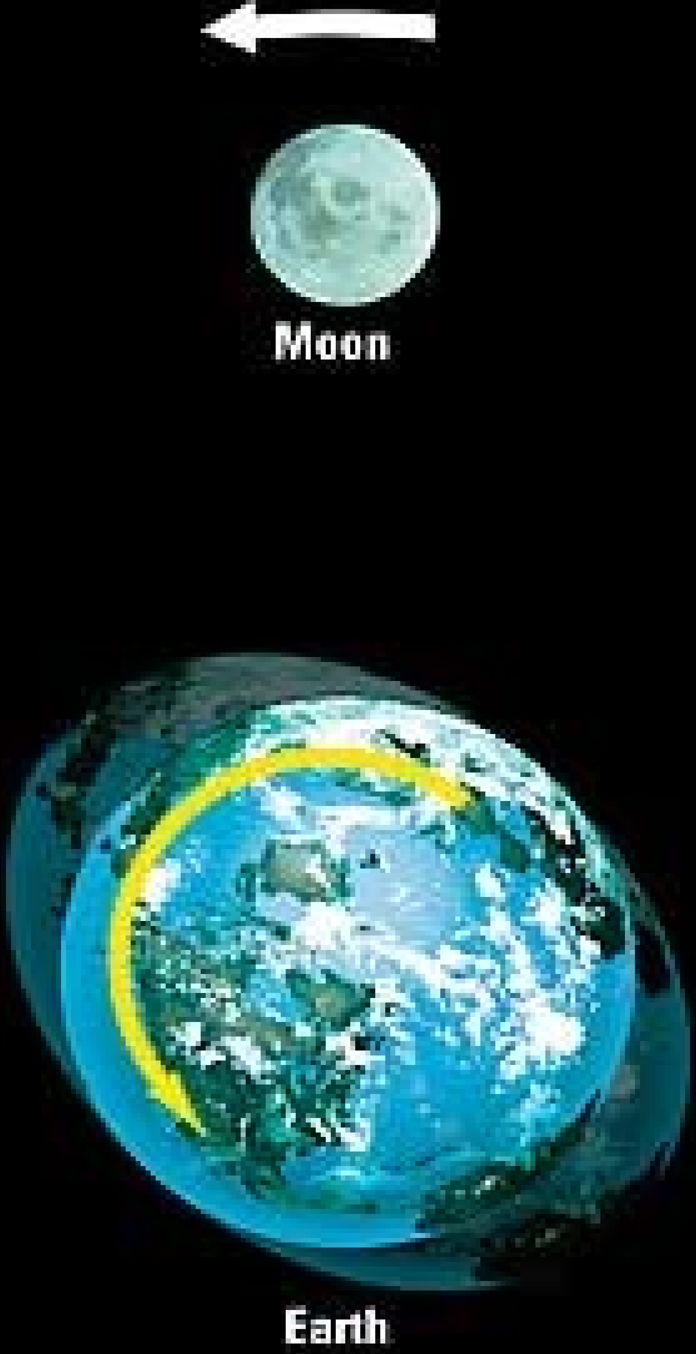


Spring tides at new and full moon, Neap tides at the First and Last Quarter phase, when sun and moon's tides work at cross-purposes


Moon
Sun
Neap
Tide
Earth
Spring
Tides

Tidal Friction…

- Now realize the Earth is rotating during all this.
- How will this affect the orientation of the tidal bulges?


Moon
Earth

Tidal Friction

- Friction between the oceans and the land will drag the water and the bulge forward of their equilibrium rotational position aligned with the moon.
- This excess mass of water will exert a gravity force on the moon, and vice versa, by Newton's 3rd Law: Forces between objects are **equal** and **opposite**

And now it’s time again for those spooky black mushroom’y things

These are Stromatolites

- Fossil stromatolites tell us how tidal friction has affected the earth and moon over geologic time.
- The algae’s grow when wet and better when warm, not when dry at low tides and less well in winter.
- Therefore, the growth rings within these fossilized intertidal zone blue-green algae colonies have encoded in them how many days in a month and how many days in a year, since growth rates depend on whether wet or dry, whether sunlit or not, and how warm it is.
- They show the Earth’s rotation has been slowing, and the transfer of that angular momentum to the moon has caused its orbit to grow

So, the day is getting longer, and the moon's orbit's getting bigger

- Conclusion: Tidal friction is transferring angular momentum **from** the Earth's <u>rotational</u> motion **to** the moon's <u>orbital</u> motion.
- So neither the Earth's spin angular momentum nor the moon's orbital angular momentum are separately conserved.
- That's because they are not isolated systems from each other.
- But the Earth-Moon system is fairly well isolated and so the angular momentum of the Earth-Moon system IS conserved.

Slowing Rotation, Growing Lunar Orbit

- Tidal friction adds about 3 milliseconds to the length of the day, each century.
- That adds up to a full hour after 100 million years (=0.1 billion years); still small compared to the 4 billion years or so the moon’s been around
- The moon’s orbit is growing by about 1 cm of extra radius per year.
- No, we won’t lose it. By the time the sun goes Red Giant, it’ll only be ~10% farther than today.

Gravity from Distributed Masses

- Ponder what gravity forces you would feel as you descended into the earth.
- Realize that every piece of matter in the Earth exerts gravity on you, and the net force on you is the combination of all those individual bits of matter pulling in all the different directions.

Your weight Decreases as you drop closer to the center, and your weight is zero at the center

13.7 – Gravitational Field Inside a Planet

- Gravitational acceleration decreases as you move towards center
- Because of mass above as well as below
- Gravity in center = zero

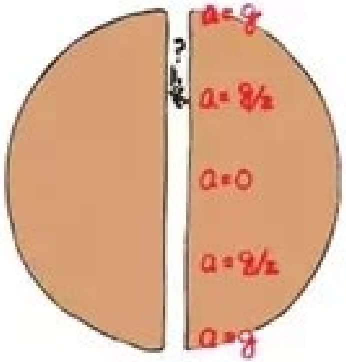


FIGURE 13.15 As you fall faster and faster into a hole bored through Earth, your acceleration diminishes because the pull of the mass above you partly cancels the pull below.

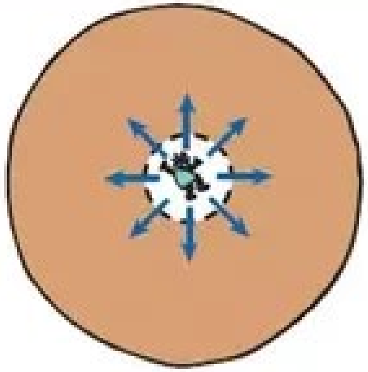
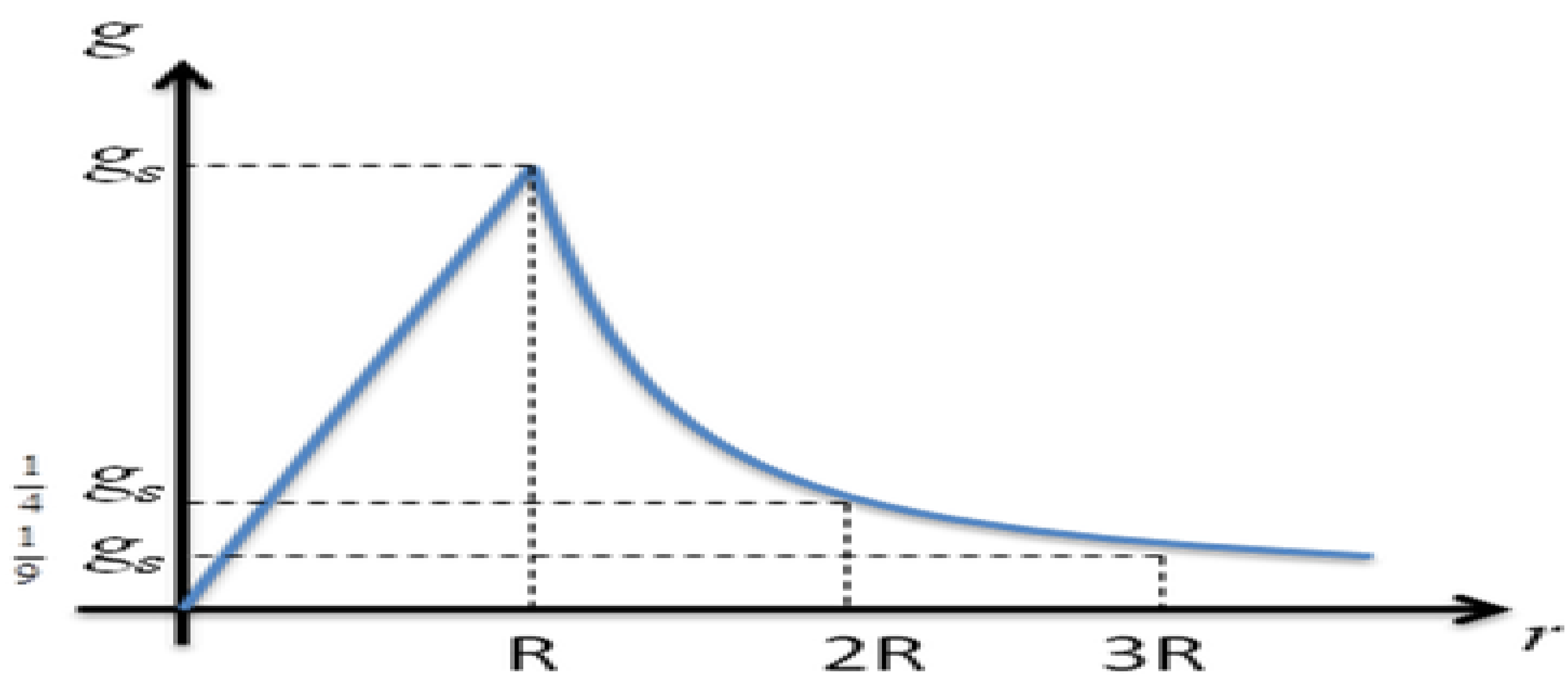


FIGURE 13.16 In a cavity at the center of Earth, your weight would be zero, because you would be pulled equally by gravity in all directions.

Imagine standing on a platform at each of these distances from the center of the Earth. This graph shows your weight. Weight=0 at Earth’s center, maximum at the surface (R) and then less as you get farther into space



A Simple Principle

- For a **spherically symmetric (like a star or planet**, or **like the dark matter halo of a galaxy)** distribution of matter, the only gravity that you will feel is the gravity due to the mass that is closer to the center than you are!
- This is a wonderful simplification, and also applies in General Relativity = our modern theory of gravity
- Check out what this means for travel inside the Earth…

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